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THERMAL CONDUCTIVITY OF A COMPOSITE MATERIAL REINFORCED BY REGULARLY DISTRIBUTED SPHEROIDAL PARTICLES

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A rigorous solution of the problem of determining the effective thermal conductivity tensor of a composite material with regularly distributed spheroidal interstitials is presented.

1. An approach to the investigation of the conductivity of composites that consists in simulating a real material by an equivalent, to some extent, regular structure has been used beginning with the pioneering works [1, 2]. A characteristic feature of this model is the possibility in principle of constructing a rigorous solution of the corresponding periodic boundary-value problem. Thus, for a composite with spherical interstitials distributed at three-dimensional lattice sites such solutions have been obtained in [3-6]. As is evident from the numerical and experimental data presented in these works, a rigorous approach to the solution of the problem is especially necessary for composites with a high concentration of highly conducting interstitials, where approximate methods do not provide adequate accuracy of the calculations. For this class of materials a decisive effect on the macroscopic properties is rendered by the interaction of the interstitials, and an adequate description of this interaction is possible only within the framework of a rigorous approach.

As is shown in [7, 8], the conductivity of a composite is affected considerably by the shape of the disperse phase particles as well. However, works containing a rigorous approach to the investigation of composites of regular structure with particles different from spherical are lacking in the literature up to now. In the present work a method for determining the effective thermal conductivity tensor of a regular composite reinforced by particles in the form of stretched spheroids that is based on a rigorous solution of a periodic three-dimensional boundary-value problem of thermal conductivity is presented. The mathematics developed might also be employed for solving a number of other problems, in particular, for investigating a composite with anisotropic phases [9].

2. The composite material considered consists of an isotropic matrix with identical particles in the form of stretched spheroids distributed in it. The particles are distributed in such a way that their centers form a three-dimensional orthogonal lattice with the lattice constants a_1, a_2, and a_3 along the Ox, Oy, and Oz axes of a Cartesian coordinate system. The semimajor axes of the spheroids are parallel to the Oz axis, and the origin of the coordinate system coincides with the center of one of the interstitials. Let us introduce a stretched spheroidal system of coordinates associated with this interstitial:

$$x + iy = d\bar{\xi}\bar{\eta}\exp(i\varphi)\,, \quad z = d\xi\eta\,, \quad \bar{\xi} = (\xi^2 - 1)^{1/2}\,, \quad \bar{\eta} = (1 - \eta^2)^{1/2}\,;$$

$$1 \le \xi < \infty\,, \quad -1 \le \eta \le 1\,, \quad 0 \le \varphi < 2\pi\,. \tag{1}$$

The spheroid surface coincides with the coordinate surface $\xi = \xi_0$, and the spheroid surface with the coordinates of its center (pa_1, qa_2, sa_3) is given by the equation $\xi_{pqs} = \xi_0$, where $(\xi_{pqs}, \eta_{pqs}, \varphi_{pqs})$ are local spheroidal coordinates corresponding to the Cartesian ones $x_{pqs} = x - pa_1$, $y_{pqs} = y - qa_2$, $z_{pqs} = z - sa_3$.

Let the composite medium be in the field of a constant external thermal flux. Then the temperature field T in the bulk of the composite satisfies the Laplace equation

$$\Delta T = 0 \tag{2}$$

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and due to the periodicity of the structure it is a quasiperiodic function of the coordinates

$$T(\mathbf{r} - a_i \mathbf{e}_i) + K_i = T(\mathbf{r}) \quad (i = 1, 2, 3), \tag{3}$$

where K_i are certain constants. The conditions of an ideal thermal contact are assumed to be satisfied on the phase interfaces:

$$[T]_{\xi_{pqs}=\xi_0} = [\mathbf{q} \cdot \mathbf{n}_{pqs}]_{\xi_{pqs}=\xi_0} = 0, \tag{4}$$

where $[f] = f_+ - f_-$ and the signs + and − denote quantities associated with the interstitials and the matrix, respectively. Thus, the problem consists in the integration of Eq. (2) under conditions of conjugation (4) and periodicity (3).

3. In constructing the solution of boundary-value problem (2)-(4) we use an approach suggested in [5] and consisting in reducing the original problem to that a composite layer of thickness a_3 containing a plane lattice of interstitials; the solution for the layer is sought in the class of doubly periodic harmonic functions. Let T_- be represented as

$$T_- = \mathbf{C} \cdot \mathbf{r} + T_1(\mathbf{r}), \tag{5}$$

where $\mathbf{C} = C_i \mathbf{e}_i$ is a constant vector; T_1 is a triply periodic solution of (2). Let us choose a layer of composite material bounded by the planes $z = h$ and $z_3 = a - h$ and containing a biperiodic system of interstitials indexed $pq0$. T_1 will be constructed as a solution that is periodic in x and y and satisfies the following conditions on the plane faces of the layer:

$$T_1 |_{z=c-h} = T_1 |_{z=h}; \quad \frac{\partial T_1}{\partial z}\Big|_{z=c-h} = \frac{\partial T_1}{\partial z}\Big|_{z=h}. \tag{6}$$

The solution in the layer is represented in the form

$$T_1 = \Gamma z + \sum_{t=0}^{\infty} \sum_{s=-t}^{t} A_{ts} F_{ts}^{*} + \sum_{m,n} (B_{mn}^{+} E_{mn}^{+} + B_{mn}^{-} E_{mn}^{-}), \tag{7}$$

where Γ, A_{ts}, B_{mn}^{+}, and B_{mn}^{-} are undetermined constants; $E_{mn}^{\pm} = \exp[\pm \gamma_{mn} z + i(\alpha_m x + \beta_n y)]$, $\alpha_m = 2\pi m / a_1$, $\beta_n = 2\pi n / a_2$, $\gamma_{mn} = \alpha_m^2 + \beta_n^2$.

In (7) F_{ts}^{*} are doubly periodic solutions of (2), external with respect to the system of poles $(pa_1, qa_2, 0)$ in the plane $z = 0$:

$$F_{ts}^{*} = \sum_{p,q} F_{ts}(\mathbf{r}_{pq0}, d) = (\mp 1)^{t+s} \sum_{m,n} \zeta_{mn}^{ts} E_{mn}^{\pm}, \quad z \lessgtr \mp d, \tag{8}$$

where

$$\zeta_{mn}^{ts} = (-1)^s \sqrt{\pi} \left(\frac{2}{\gamma_{mn} d} \right)^{t+1/2} I_{t+1/2}(\gamma_{mn} d) \frac{2\pi}{a_1 a_2} \gamma_{mn}^{t-s-1} (\beta_n - i\alpha_m)^s,$$

$$t = 1, 2, \ldots, \quad |s| \le t; \quad \zeta_{00}^{10} = \frac{8\pi}{3 a_1 a_2}, \quad \zeta_{00}^{ts} = 0 \quad (t \ne 1, \quad s \ne 0);$$

$F_{ts}(\mathbf{r}, d)$ are external particular solutions of (2) in stretched spheroidal coordinates:

$$F_{ts}(\mathbf{r}, d) = \left(\frac{2}{d} \right)^{t+1} \frac{(t-s)!}{(t+s)!} Q_t^s(\xi) P_t^s(\eta) \exp(is\varphi). \tag{9}$$

The procedure for constructing the system of functions (8) and representing them by Fourier series is, on the whole, similar to that described in [5]. The second part of expression (7) is the representation of the general doubly periodic solution of (2) for the layer by a double Fourier series.

Let us choose the constants B^+_{mn} and B^-_{mn} in such a way that they ensure that the conditions of periodicity of the solution T_1 in z are satisfied. For that we substitute (7) into conditions (6) with account for the representation of F^*_{ts} by Fourier series (8). By equating coefficients of the same harmonics on both sides of the equations we come to the following relations:

$$B^{\pm}_{mn} = \Delta_{mn} \sum_{t=1}^{\infty} \sum_{s=-t}^{t} (\mp 1)^{t+s} A_{ts} \zeta^{ts}_{mn} ,$$
$$\Delta_{mn} = [\exp (\gamma_{mn} c) - 1]^{-1} , \quad \Gamma = - \frac{16\pi}{3a_1 a_2 a_3} A_{10} . \tag{10}$$

A second group of relations is obtained by satisfying the conditions of conjugation in the matrix and the interstitials. It should be noted that due to the periodicity of the problem it is sufficient to satisfy the conditions (4) for the interstitial with $p = q = s = 0$. The temperature in this interstitial is represented by the series

$$T_+ = \sum_{t=0}^{\infty} \sum_{s=-t}^{t} D_{ts} f_{ts} (\mathrm{r} , d) , \tag{11}$$

where

$$f_{ts} (\mathrm{r} , d) = \left(\frac{d}{2} \right)^t \frac{(t - s)\,!}{(t + s)\,!} P^s_t (\xi) P^s_t (\eta) \exp (i s \varphi)$$

are internal particular solutions of (2).

Transformation of the Fourier series in (7) to spheroidal coordinates is realized by means of the relations

$$E_{mn} = \sum_{t=0}^{\infty} \sum_{s=-t}^{t} (\pm 1)^{t+s} M^{ts}_{mn} f_{ts} (\mathrm{r} , d) , \tag{12}$$

where

$$M^{ts}_{mn} = \sqrt{\pi} (t + 1/2) \left(\frac{2}{\gamma_{mn} d} \right)^{t+1/2} I_{t+1/2} (\gamma_{mn} d) \gamma^{t+s}_{mn} (\beta_n - i \alpha_m)^{-s} .$$

The representation of the functions F^*_{ts} in the coordinates (ξ, η, φ) is based on the use of the following addition theorems for particular solutions of the Laplace equation in spheroidal coordinates:

$$F_{ts} (\mathrm{r}_{pqs} , d) = \sum_{k=0}^{\infty} \sum_{l=-k}^{k} N^{s-l}_{tk} f_{kl} (\mathrm{r} , d) , \quad \mathrm{r}_{pqs} = \mathrm{r} + \mathbf{R}_{pqs} , \tag{13}$$

where

$$N^{\mu}_{tk} = N^{\mu(1)}_{tk} = (- 1)^{k+\mu} (k + 1/2) \pi \sum_{\nu=0}^{\infty} \left(\frac{d}{2} \right)^{2\nu} \times$$
$$\times \frac{(t+k + \nu + 2)_{\nu} Y^{\mu}_{t+k+2\nu} (\mathbf{R}_{pqs})}{\nu\,! \Gamma (t + \nu + 3/2) \Gamma (k + \nu + 3/2)} , \quad R_{pqs} > 2d ; \tag{14}$$

R_{pqs} is arbitrary at $\tilde{d}/d > 1$; $Y_t^s(\mathbf{r}) = r^t/(t+s)! \cdot P_t^s(\eta) \exp(is\varphi)$ are particular solutions of the Laplace equation in spheroidal coordinates; $(n)_m = n(n+1) \ldots (n+m-1)$. The transition formula (13), (14) is of independent interest, similar relations being known only for the case of coaxial spheroids [10, 11]; the expression $N_{tk}^{\mu(1)}$ can be obtained by taking the limit in the addition theorem for the spheroidal wave functions [12]. Substituting (13) into (8) we obtain

$$N_{tk}^{\mu} = N_{tk}^{\mu(2)} = (-1)^k (k+1/2)\sqrt{\pi} \sum_{\nu=0}^{\infty} \left(\frac{d}{2}\right)^{2\nu} \times (t+k+2\nu+1/2) \sum_{j=0}^{\nu} \left(\frac{\tilde{d}}{d}\right)^{2\nu} \frac{(-1)^j}{j!\,(\nu-j)!} \times$$

$$\times \frac{(t+k+\nu-j)_\nu \, \Gamma(t+k+2\nu-j+1/2)}{\Gamma(t+\nu-j+3/2)\,\Gamma(k+\nu-j+3/2)} F_{t+k+2\nu}^{\mu}(\mathbf{R}_{pqs}, \tilde{d}),$$

$$F_{ts}^{*} = F_{ts}(\mathbf{r}, d) + \sum_{k=0}^{\infty} \sum_{l=-k}^{k} N_{tk}^{s-l*} f_{kl}(\mathbf{r}, d), \tag{15}$$

where

$$N_{tk}^{s-l*} = \sum_{\substack{p,q \\ P_{pq0}<2d}}' N_{tk}^{s-l(2)} + \sum_{\substack{p,q \\ P_{pq0}>2d}} N_{tk}^{s-l(1)},$$

the prime denotes the absence of the term with $p = q = 0$. Taking into account (14) the calculation of N_{tk}^{s-l*} is reduced to summing up series of the form $\sum_{p,q}' Y_\nu^\mu(\mathbf{R}_{pq\,0})$, for which techniques for improving convergence have been highly developed [13].

Considering (10), (12), and (15) the expression for T_- takes the form

$$T_- = 2\left[(C_1 - iC_2) f_{11} - (C_1 - iC_2) f_{1,-1} + (\Gamma + C_3) f_{10}\right] +$$

$$+ \sum_{t=1}^{\infty} \sum_{s=-t}^{t} A_{ts} F_{ts} + \sum_{t=0}^{\infty} \sum_{s=-t}^{t} f_{ts} \sum_{k=1}^{\infty} \sum_{l=-k}^{k} A_{kl} d_{kt}^{ls}, \tag{16}$$

where

$$d_{kt}^{ls} = N_{tk}^{s-l*} + [(-1)^{k+1} + (-1)^{t+s}] \sum_{m,n} M_{mn}^{ts} \zeta_{mn}^{kl} \Delta_{mn}.$$

Substituting (16) and (11) into the first of conditions (4) and equating the coefficients of the same harmonics one obtains

$$G + A_{ts}\left(\frac{2}{d}\right)^{2t+1} \frac{Q_t^s(\xi_0)}{P_t^s(\xi_0)} + \sum_{k=1}^{\infty} \sum_{l=-k}^{k} A_{kl} d_{kt}^{ls} = D_{ts},$$

where $G = 2\delta_t^1[(C_1 - iC_2)\delta_s^1 - (C_1 - iC_2)\delta_s^{-1} + (\Gamma_3 + C_{10})\delta_s^0]$.

Similarly, from the second condition of (4) one finds

$$G + A_{ts}\left(\frac{2}{d}\right)^{2t+1} \frac{Q_t^{s\prime}(\xi_0)}{P_t^{s\prime}(\xi_0)} + \sum_{k=1}^{\infty} \sum_{l=-k}^{k} A_{kl} d_{kt}^{ls} = \kappa D_{ts}.$$

Eliminating the unknowns D_{ts} from the last two equations one arrives at the following infinite system of linear algebraic equations for A_{ts}:

$$\frac{A_{ts}}{(\kappa-1)}\left(\frac{2}{d}\right)^{2t+1}\left[\kappa\frac{Q_t^s(\xi_0)}{P_t^s(\xi_0)}-\frac{Q_t^{s'}(\xi_0)}{P_t^{s'}(\xi_0)}\right]+\sum_{k=1}^{\infty}\sum_{l=-k}^{k}A_{kl}\,d_{kt}^{ls}=-G. \tag{17}$$

The final expressions for D_{ts} are

$$D_{ts}=\frac{A_{ts}}{(\kappa-1)}\left(\frac{2}{d}\right)^{2t+1}\left[\frac{Q_t^s(\xi_0)}{P_t^s(\xi_0)}-\frac{Q_t^{s'}(\xi_0)}{P_t^{s'}(\xi_0)}\right]. \tag{18}$$

Analysis of the coefficients (17) with account for expressions (14), (16) and the estimates of the double series of [5] proves that it is a system of normal type, provided adjacent interstitials do not touch ($2d\xi < a_3$, $2d\xi^- < min(a_1, a_2)$) and, consequently, that it is possible to obtain its solution in an arbitrary approximation by the reduction method.

4. The tensor of the effective composite material thermal conductivity coefficients Λ of the composite material is determined from the relations

$$\langle \mathrm{q}\rangle=-\Lambda\langle\nabla T\rangle,\quad \Lambda=\left\{\lambda_{ij}\right\}; \tag{19}$$

where $\langle f\rangle = 1/V\int_V f dV$, V is a representative volume of the composite. Due to the regularity of the structure it may be a parallelepiped with sides a_1, a_2 and a_3 that contains one interstitial. For calculating $\langle\nabla T\rangle$ the gradient theorem is used:

$$\int_V \nabla\Phi(\mathrm{r})\,dV=\int_S \Phi(\mathrm{r})\,dS.$$

We obtain (20)

$$V\langle\nabla T\rangle=\int_V \nabla T\,dV=\int_{V_-}\nabla T_-\,dV+\int_{V_+}\nabla T_+\,dV=\int_\Sigma T_-\,dV-\int_{S_-}T_-\,dS+\int_{S_+}T_+\,dS,$$

where $\sum$ is the surface of the plane faces of the parallelepiped; S is the surface of the interstitial. Taking into account equality of the temperatures at the phase interface and the periodicity of the solution in the matrix it is easily found that $\langle\nabla T\rangle = \mathbf{C}$, i. e., the vector $\mathbf{C}$ has the meaning of the average temperature gradient. Averaging of the flux q gives

$$V\langle\mathrm{q}\rangle=\int_{V_-}\mathrm{q}_-\,dV+\int_{V_+}\mathrm{q}_+\,dV=-\lambda_-\int_{V_-}\nabla T_-\,dV-\lambda_+\int_{V_+}\nabla T_+\,dV.$$

Considering (20), $V\langle q\rangle = -\lambda_-\langle\nabla T\rangle - [\lambda]\int_S T_+ dS$.

Calculating the integral over the spheroid surface $\xi = \xi_0$, one finds

$$-\langle\mathrm{q}\rangle=\lambda_-\langle\nabla T\rangle+[\lambda]\frac{f}{2}\mathrm{Re}\left[\mathrm{e}_3 D_{10}+(\mathrm{e}_1-i\,\mathrm{e}_2)D_{11}\right]. \tag{21}$$

Since the components of the vector $\mathbf{C}$ enter the right-hand side of system (17) as parameters, relations (17), (19), and (21) are sufficient for determining all the components of the effective thermal conductivity tensor. In particular, analysis of the system gives $\lambda_{ij} = 0$ for $i \neq j$ which corresponds to the composite material structure considered. For the nonzero components of the tensor Λ with account for (18) one obtains the expressions

$$\frac{\lambda_{33}}{\lambda_0}=1-\frac{4f}{C_3R}A_{10},\quad \frac{\lambda_{11}}{\lambda_0}=1+\frac{8f}{C_1R}\mathrm{Re}\,A_{11},\quad \frac{\lambda_{22}}{\lambda_0}=1-\frac{8f}{C_2R}\mathrm{Im}\,A_{11}, \tag{22}$$

TABLE 1. Dependence of $\lambda_{11} = \lambda_{22}$ and λ_{33} on κ and f for a Composite with $R = 2$, $p_1/a_1 = p_2/a_2 = p_3/a_3$

f	κ					
	0	2	10	20	100	1000
0.1	0.845	1.074	1.215	1.246	1.275	1.282
	0.876	1.084	1.331	1.410	1.496	1.520
0.2	0.712	1.153	1.485	1.565	1.643	1.663
	0.752	1.168	1.668	1.832	2.013	2.062
0.3	0.591	1.239	1.830	1.994	2.162	2.206
	0.630	1.255	2.054	2.333	2.652	2.744
0.4	0.478	1.331	2.032	2.629	3.000	3.104
	0.510	1.345	2.532	2.991	3.552	3.716
0.5	0.367	1.430	2.063	3.875	5.161	5.635
	0.392	1.441	3.180	3.978	5.076	5.422

where $R = \bar{\xi}_0/\xi_0$ is the ratio of the spheroid axes. Only the first unknowns of system (17) enter relations (22), which ensures sufficient accuracy of the results when keeping even a small number of equations in the system.

5. Some results obtained in implementing the method described on a computer are presented here. The computation was carried out by keeping unknowns up to the index $t = 13$ in the infinite system, which ensured satisfaction of boundary conditions (4) with an error less than 1% over the whole range of parameter changes considered. The method is simple enough from the point of view of numerical realization, the most complicated matter being the calculation of the coefficients of the matrix of the system since they are double series. The computation being prepared and performed properly, it takes no more than 1-2 min of AT 286/287 computer time to solve the problem. Moreover, the above sums, once calculated for given ratios of the structure parameters f/a_1, a_2/a_1, and a_3/a_1, may be used for calculating the macroscopic thermal conductivity of the composite for arbitrary values of f and κ.

A characteristic feature of the material considered is anisotropy caused both by the structure of the lattice of interstitials and by their shape. Moreover, even in the simplest case where the lattice constants a_i are proportional to the spheroid semiaxes p_i, anisotropy turns out to be rather substantial. Thus, Table 1 gives the components of the tensor Λ calculated by the method described above as a function of the ratio of properties κ and the volume share of interstitial f with $R = 2$. In all the tables the upper values correspond to $\lambda_{11}/\lambda_0 = \lambda_{22}/\lambda_0$, and the lower ones to λ_{33}/λ_0. As is seen from the table, there is anisotropy growth with increase in κ; on the other hand, at large κ and a concentration near the maximum ($f = 0.5$) the composite is practically isotropic.

It is of interest to compare the results obtained by us with those of calculations carried out by other methods. Thus, formulas for calculating the components λ_{11} and λ_{33} of a composite with unidirectional spheroids as a filler are presented in [7]. Results of a comparison for some parameter values are given in Table 2. Since in [7] the structure of the composite is not patently stated, the condition of proportionality of p_i and a_i has been chosen, as in the previous case. The table shows that at $\kappa = 0$ (a porous material) there is rather good agreement for λ_{33} over practically the whole range of considered R and f, whereas for λ_{11} already at $R = 3.0$ and $f = 0.3$ the divergence is about 20%, and with growth of these parameters, it increases. On the other hand, for a composite material ($\kappa = 20$) the best agreement is for λ_{11}; thus at $f = 0.4$ and $R = 5$ the divergence is 22%, whereas the compared values λ_{33} differ by more than twofold. For a highly nonuniform material ($\kappa = 1000$) the results of the calculations are comparable only at a low (≤ 0.1) concentration of the filler and moderate values of R. So, the analysis presented makes it possible to establish the limits of applicability of approximate approaches and to ascertain the class of materials in predicting the properties of which a rigorous approach is necessary: highly filled, highly nonuniform (e.g., diamond-polymer) composites are such.

It should be noted that the comparison described is given for just one relation between the lattice parameters and the particle geometry; naturally, its variation affects the composite properties. As is seen from Table 3 (and

TABLE 2. Comparison of Values of λ_{11} and λ_{33} Calculated by Means of Formula (22) and by the Method of [7]

R	κ	$f=0.1$		$f=0.2$		$f=0.3$		$f=0.4$		$f=0.5$	
		(22)	[7]	(22)	[7]	(22)	[7]	(22)	[7]	(22)	[7]
1.5	0	0.850	0.838	0.718	0.678	0.598	0.519	0.486	0.365	0.376	0.216
		0.870	0.869	0.743	0.736	0.621	0.600	0.502	0.450	0.384	0.310
	20	1.258	1.295	1.588	1.801	2.026	2.700	2.670	4.173	3.673	6.211
		1.353	1.460	1.739	2.249	2.200	3.562	2.800	5.461	3.397	7.740
	1000	1.298	1.365	1.649	2.242	2.252	7.198	3.165	83.77	5.717	–
		1.431	1.642	1.914	3.375	2.518	13.92	3.365	146.1	4.765	
3.0	0	0.840	0.820	0.705	0.643	0.584	0.470	0.470	0.305	0.358	0.155
		0.882	0.887	0.759	0.772	0.638	0.654	0.518	0.529	0.398	0.387
	20	1.235	1.254	1.546	1.676	1.967	2.399	2.594	3.610	3.813	5.431
		1.488	1.804	1.957	3.043	2.519	4.751	3.272	6.826	4.434	9.069
	1000	1.268	1.311	1.673	2.055	2.168	5.797	3.054	47.82	5.567	–
		1.655	2.576	2.281	7.596	3.088	35.36	4.297	209.2	6.578	
5.0	0	0.837	0.811	0.701	0.624	0.579	0.415	0.464	0.267	0.348	0.115
		0.885	0.894	0.764	0.786	0.643	0.675	0.522	0.560	0.403	0.426
	20	1.229	1.238	1.538	1.620	1.956	2.256	2.578	3.328	3.375	5.021
		1.537	2.136	2.037	3.664	2.644	5.509	3.471	7.564	4.766	9.700
	1000	1.259	1.295	1.621	1.992	2.145	4.944	3.025	28.41	5.545	–
		1.816	4.503	2.554	16.27	3.557	66.38	5.170	246.0	8.605	

TABLE 3. Dependence of the Thermal Conductivity of a Composite (κ = 1000) with a Cubic Lattice of Interstitials on Their Geometry, $f = 0.1$

κ	R					
	1.0	1.25	1.50	1.75	2.0	2.25
0	0.857	0.851	0.846	0.841	0.838	0.834
		0.868	0.875	0.881	0.885	0.888
2	1.052	1.075	1.074	1.073	1.072	1.072
		1.080	1.084	1.086	1.089	1.091
10	1.202	1.228	1.217	1.209	1.202	1.197
		1.282	1.325	1.369	1.416	1.471
20	1.255	1.263	1.248	1.238	1.229	1.223
		1.338	1.401	1.471	1.550	1.650
1000	1.332	1.304	1.284	1.271	1.260	1.252
		1.410	1.506	1.662	1.768	1.983

from a comparison with the above mentioned data, as well), such an effect is very substantial. Therefore in simulating a real material it is important to choose parameters that would take into account its structure to the utmost.

Thus, in this investigation the problem of determining the effective thermal conductivity tensor of a granular composite material reinforced by regularly distributed spheroidal particles has been solved in a rigorous formulation. It should also be noted that the supposition, initially adopted, of the orthogonality of the lattice of interstitials is not fundamental since all the results are easily generalized to the case of nonorthogonal structures.

An investigation of the thermal conductivity of a composite with a filler in the form of compressed spheroids may be carried out in a similar way; the problem consists only in establishing the relations analogous to (8), (12), and (13).

NOTATION

T, temperature, a_1, a_2, a_3, lattice constants; $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, Cartesian unit vectors; (x, y, z), Cartesian coordinates; (ξ, η, φ), stretched spheroidal coordinates; λ, thermal conductivity; $\kappa = \lambda_+/\lambda_-$; $\mathbf{q} = -\lambda\nabla T$, thermal flux vector; $f = V_+/(V_+ + V_-)$, volume share of interstitials; $\mathbf{n}_{pqs}$, unit vector of the normal to the surface of the pqs-th interstitial; $2d$, interfocus distance; $p_1 = p_2 = d\bar{\xi}_0$, $p_3 = d\xi_0$, semiaxes of a spheroid; $I_\nu(z)$, modified Bessel function of the first kind; $P_t^s(z)$, $Q_t^s(z)$, associated Legendre functions of the first and second kind, respectively; $\Gamma(z)$, gamma-function; δ_i^j, Kronecker symbol.

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